Hoofdstuk 5: Werken met formules

5.1 Stelsels vergelijkingen

<u>Opgave 1:</u>
a. $60 - 10 \cdot 1{,}6 = 44$

   $\dfrac{44}{0{,}4} = 110$ dus 110 bolletjes

b. $60 - 90 \cdot 0{,}4 = 24$

   $\dfrac{24}{2{,}6} = 15$ dus 15 broden

c. $1{,}6x + 0{,}4y = 60$

<u>Opgave 2:</u>
a. $15x + 12y = 2520$
   $12y = -15x + 2520$
   $y = -1{,}25x + 210$

b. $3p - 2q = 16\tfrac{1}{2}$
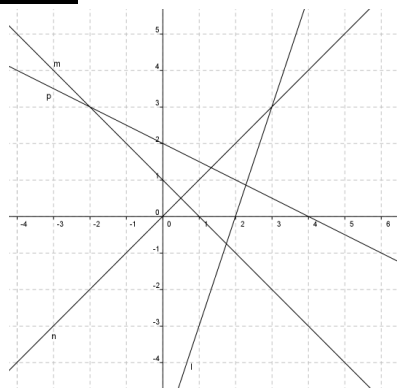 $3p = 2q + 16\tfrac{1}{2}$
   $p = \tfrac{2}{3}q + 5\tfrac{1}{2}$

c. $5a - 2b = 16$
   $-2b = -5a + 16$
   $b = 2{,}5a - 8$

<u>Opgave 3:</u>
a.

b. $l:\quad 3x - y = 16$
   $\qquad -y = -3x + 16$
   $\qquad y = 3x - 16$ dus $rc_l = 3$
   $m:\quad x + y = 1$
   $\qquad y = -x + 1$ dus $rc_m = -1$
   $n:\quad x - y = 0$
   $\qquad -y = -x$
   $\qquad y = x$ dus $rc_n = 1$
   $p:\quad x + 2y = 4$

$$2y = -x + 4$$
$$y = -\tfrac{1}{2}x + 2 \text{ dus } rc_p = -\tfrac{1}{2}$$

Opgave 4:
a. snijpunt x-as: $y = 0$ dus
$$4x = 24$$
$$x = 6 \text{ dus } (6,0)$$
   snijpunt y-as: $x = 0$ dus
$$-3y = 24$$
$$y = -8 \text{ dus } (0,-8)$$

b. $4 \cdot 8 - 3 \cdot 3 = 23$ dus A niet
$4 \cdot 18 - 3 \cdot 16 = 24$ dus B wel
$4 \cdot -30 - 3 \cdot -48 = 24$ dus C wel

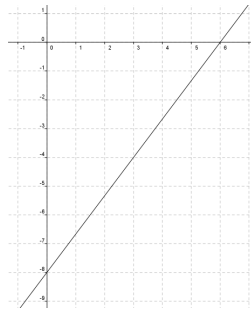c. $4 \cdot 16 - 3p = 24$
$$64 - 3p = 24$$
$$-3p = -40$$
$$p = 13\tfrac{1}{3}$$

d. $4q - 3 \cdot 48 = 24$
$$4q - 144 = 24$$
$$4q = 168$$
$$q = 42$$

Opgave 5:
a. $l: \ 3x - 4y = 7$
$$-4y = -3x + 7$$
$$y = \tfrac{3}{4}x - 1\tfrac{3}{4} \text{ dus } rc_l = \tfrac{3}{4}$$
   $m: \ 3x - 4y = -8$
$$-4y = -3x - 8$$
$$y = \tfrac{3}{4}x + 2 \text{ dus } rc_l = \tfrac{3}{4}$$

b. $rc_l = \tfrac{3}{4}$

c. $3 \cdot 5 - 4 \cdot 1 = 11$ dus $c = 11$

d. $3x - 4y = c$ door $(3,-1)$
$$3 \cdot 3 - 4 \cdot -1 = 13 \text{ dus } c = 13$$
$$k: \ 3x - 4y = 13$$

Opgave 6:
$2x + y = c$ door $(5,8)$
$2 \cdot 5 + 8 = 18$ dus $c = 18$
$2x + y = 18$

Opgave 7:
$12x + 4y = 242,40$

Opgave 8:
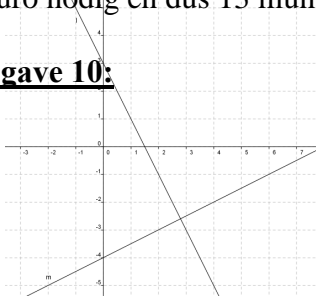Stel x kaartjes van 10 euro en y kaartjes van 15 euro
$10x + 15y = 4300$

Opgave 9:
Als je alleen 50 munten van 1 euro hebt, dan kom je 37 euro tekort, dus heb je 37 munten van 2 euro nodig en dus 13 munten van 1 euro

Opgave 10:
a.

b.　(2,–1)

c.　(2,–1)

Opgave 11:

a.　$\begin{cases} 5x - 4y = -8 \\ -x + 4y = -12 \end{cases} +$

$\quad 4x = -20$

$\quad x = -5$

$\quad 5 + 4y = -12$

$\quad 4y = -17$

$\quad y = -4\frac{1}{4}$

$\quad$ dus $(-5, -4\frac{1}{4})$

b.　$\begin{cases} -2x + y = 7 \\ -2x + 3y = -1 \end{cases} -$

$\quad -2y = 8$

$\quad y = -4$

$\quad -2x - 4 = 7$

$\quad -2x = 11$

$\quad x = -5\frac{1}{2}$

$\quad$ dus $(-5\frac{1}{2}, -4)$

c.　$\begin{cases} -x - 3y = -8 \\ -2x + 3y = -1 \end{cases} +$

$\quad -3x = -9$

$\quad x = 3$

$\quad -3 - 3y = -8$

$\quad -3y = -5$

$$y = 1\tfrac{2}{3}$$

dus $(3, 1\tfrac{2}{3})$

Opgave 12:

a. $5x - y = 23$ dus nee

b. $x - 7y = -9$ dus nee

Opgave 13:

a. $\begin{cases} 3x + 5y = -7 & \vert \times 1 \\ 2x + y = 0 & \vert \times 5 \end{cases}$

$\begin{cases} 3x + 5y = -7 \\ \underline{10x + 5y = 0} \quad - \end{cases}$

$-7x = -7$

$x = 1$

$3 + 5y = -7$

$5y = -10$

$y = -2$

dus $(1, -2)$

b. $\begin{cases} 2x - 4y = 6 & \vert \times 1 \\ 3x - y = 19 & \vert \times 4 \end{cases}$

$\begin{cases} 2x - 4y = 6 \\ \underline{12x - 4y = 76} \quad - \end{cases}$

$-10x = -70$

$x = 7$

$14 - 4y = 6$

$-4y = -8$

$y = 2$

dus $(7, 2)$

c. $\begin{cases} 4x + y = 13 & \vert \times 2 \\ x - 2y = 1 & \vert \times 1 \end{cases}$

$\begin{cases} 8x + 2y = 26 \\ \underline{x - 2y = 1} \quad + \end{cases}$

$9x = 27$

$x = 3$

$3 - 2y = 1$

$-2y = -2$

$y = 1$

dus $(3, 1)$

Opgave 14:

a. $\begin{cases} 5x + 2y = 69 \\ x + 3y = -7 \end{cases} \quad \begin{array}{l} \times 1 \\ \times 5 \end{array}$

$\begin{cases} 5x + 2y = 69 \\ \underline{5x + 15y = -35} \quad - \end{cases}$

$-13y = 104$

$y = -8$

$x - 24 = -7$

$x = 17$

dus $(17,-8)$

b. $\begin{cases} 2x - 5y = -19 \\ 5x + 4y = 35 \end{cases} \quad \begin{array}{l} \times 4 \\ \times 5 \end{array}$

$\begin{cases} 8x - 20y = -76 \\ \underline{25x + 20y = 175} \quad + \end{cases}$

$33x = 99$

$x = 3$

$6 - 5y = -19$

$-5y = -25$

$y = 5$

dus $(3,5)$

c. $\begin{cases} 0,8x + 0,2y = 1 \\ 0,3x - 0,3y = 1,5 \end{cases} \quad \begin{array}{l} \times 3 \\ \times 2 \end{array}$

$\begin{cases} 2,4x + 0,6y = 3 \\ \underline{0,6x - 0,6y = 3} \quad + \end{cases}$

$3x = 6$

$x = 2$

$1,6 + 0,2y = 1$

$0,2y = -0,6$

$y = -3$

dus $(2,-3)$

Opgave 15:

a.

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b. $\begin{cases} 2x + 3y = 15 \\ x - 2y = 1 \end{cases} \quad \begin{matrix} \times 2 \\ \times 3 \end{matrix}$

$\begin{cases} 4x + 6y = 30 \\ \underline{3x - 6y = 3} \quad + \end{cases}$

$7x = 33$

$x = 4\frac{5}{7}$

$4\frac{5}{7} - 2y = 1$

$-2y = -3\frac{5}{7}$

$y = 1\frac{6}{7}$

dus $(4\frac{5}{7}, 1\frac{6}{7})$

Opgave 16:

$\begin{cases} 3x - 2y = -12 \\ x + 4y = 38 \end{cases} \quad \begin{matrix} \times 2 \\ \times 1 \end{matrix}$

$\begin{cases} 6x - 4y = -24 \\ \underline{x + 4y = 38} \quad + \end{cases}$

$7x = 14$

$x = 2$

$2 + 4y = 38$

$4y = 36$

$y = 9$

dus $(2,9)$

Opgave 17:

$\begin{cases} 2x + 3y = 12 \\ y = 4x - 10 \end{cases}$

$2x + 3(4x - 10) = 12$

$2x + 12x - 30 = 12$

$14x = 42$

$x = 3$

$y = 4 \cdot 3 - 10 = 2$

dus $(3,2)$

Opgave 18:

a. $\begin{cases} 2x + 2y = 9 \\ y = 4x - 3 \end{cases}$

$2x + 2(4x - 3) = 9$

$2x + 8x - 6 = 9$

$10x = 15$

$x = 1\frac{1}{2}$

$y = 4 \cdot 1\frac{1}{2} - 3 = 3$

dus $(1\frac{1}{2}, 3)$

b.
$$\begin{cases} y = \frac{1}{2}x + 1 \\ 3x + 6y = 8 \end{cases}$$
$3x + 6 \cdot (\frac{1}{2}x + 1) = 8$
$3x + 3x + 6 = 8$
$6x = 2$
$x = \frac{1}{3}$
$y = \frac{1}{2} \cdot \frac{1}{3} + 1 = 1\frac{1}{6}$
dus $(\frac{1}{3}, 1\frac{1}{6})$

c.
$$\begin{cases} x = 5y - 3 \\ 3x + 4y = 29 \end{cases}$$
$3 \cdot (5y - 3) + 4y = 29$
$15y - 9 + 4y = 29$
$19y = 38$
$y = 2$
$x = 5 \cdot 2 - 3 = 7$
dus $(7,2)$

5.2 Vergelijkingen gebruiken

Opgave 19:

a. voor $x = 3$ geldt: $y = 5 \cdot 3 - 7 = 8$

b. $x = 2$ en $y = 6$ geeft: $2a + b = 6$

c. $-a + b = 4$

Opgave 20:

$$\begin{cases} a + c = 8 \\ 4a + c = 17 \quad - \end{cases}$$

$$-3a = -9$$
$$a = 3$$
$$c = 5$$

Opgave 21:

$$\begin{cases} 2a + b = 8 & \quad \times 2 \\ 2b + a = 8 & \quad \times 1 \end{cases}$$

$$\begin{cases} 4a + 2b = 16 \\ a + 2b = 8 \quad - \end{cases}$$

$$3a = 8$$
$$a = 2\tfrac{2}{3}$$
$$5\tfrac{1}{3} + b = 8$$
$$b = 2\tfrac{2}{3}$$

Opgave 22:

a.
$$\begin{cases} 4 + 2p + q = -1 \\ 4p - q = -1 \quad + \end{cases}$$

$$4 + 6p = -2$$
$$6p = -6$$
$$p = -1$$
$$-4 - q + -1$$
$$-q = 3$$
$$q = -3$$

b. $x^2 - x - 3 = -2x + 3$

$$x^2 + x - 6 = 0$$
$$(x + 3)(x - 2) = 0$$
$$x = -3 \quad \vee \quad x = 2$$
$$y = 9 \qquad \quad y = -1$$

dus $(-3, 9)$

Opgave 23:

door $(0,4)$ dus $c = 4$

$$\begin{cases} 4a - 2b + 4 = -10 & |\times 3 \\ 9a + 3b + 4 = -5 & |\times 2 \end{cases}$$

$$\begin{cases} 12a - 6b + 12 = -30 \\ \underline{18a + 6b + 8 = -10} \quad + \end{cases}$$

$30a + 20 = -40$

$30a = -60$

$a = -2$

$-8 - 2b + 4 = -10$

$-2b = -6$

$b = 3$

Opgave 24:
a.

b. $DP = AD - AP = 20 - 5 = 15$

$AD = \sqrt{15^2 - 5^2} = \sqrt{200} = 14,1$

c. $DP = 20 - 8 = 12$

$AD = \sqrt{12^2 - 8^2} = \sqrt{80} = 8,9$

d. $DP = AD - AP = 20 - x$

$AD = AP = x$

e. $AD^2 + AP^2 = DP^2$

$x^2 + x^2 = (20 - x)^2$

f. $\left.\begin{array}{l} y_1 = 2x^2 \\ y_2 = (20 - x)^2 \end{array}\right\}$ de optie intersect geeft: $x = 8,3$

g. $AP = 8,3\,\text{cm}$

Opgave 25:
a. $PQ = AB - 2 \cdot rand = 8 - 2x$

b. $PS = 6 - 2x$

c. $Opp(PQRS) = (8 - 2x)(6 - 2x)$

d. $(8 - 2x)(6 - 2x) = 30$

$\left.\begin{array}{l} y_1 = (8 - 2x)(6 - 2x) \\ y_2 = 30 \end{array}\right\}$ de optie intersect geeft: $x = 0,72 \quad \vee \quad x = 6,28\,(\text{vervalt})$

dus $x = 72\,\text{cm}$

Opgave 26:
$Opp(PQRS) = 25$ dus $PQ = 5$

neem $BP = x$ dan $BQ = 7 - x$

in ΔBPQ geldt: $BP^2 + BQ^2 = PQ^2$

$x^2 + (7 - x)^2 = 25$

$\left.\begin{array}{l} y_1 = x^2 + (7 - x)^2 \\ y_2 = 25 \end{array}\right\}$ de optie intersect geeft: $x = 3 \quad \vee \quad x = 4$

dus $AP = 3 \quad \vee \quad AP = 4$

Opgave 27:

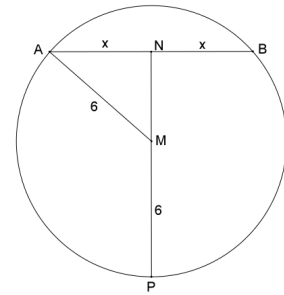als $AN = x$ dan $AB = 2x$

$NP = 2 \cdot AB = 2 \cdot 2x = 4x$

$MN = NP - MP = 4x - 6$

$AN^2 + MN^2 = AM^2$

$(4x - 6)^2 + x^2 = 36$

$\left.\begin{array}{l} y_1 = (4x-6)^2 + x^2 \\ \\ y_2 = 36 \end{array}\right\}$ de optie intersect geeft: $x = 0 \quad \vee \quad x = 2,82$

$AB = 2x = 5,65$

Opgave 28:

$Opp = 5 \cdot 4 + \frac{1}{2} \cdot 9 \cdot 5 - \frac{1}{2} \cdot 4 \cdot 3 = 36\frac{1}{2}$
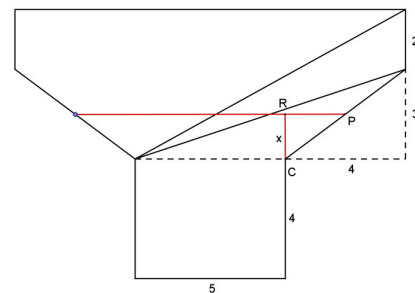
$Opp(CPQH) = 36\frac{1}{2} - 5 \cdot 4$

stel $CR = x$ dan geldt:

$\dfrac{x}{3} = \dfrac{PR}{4}$

$PR = \dfrac{4x}{3} = \frac{4}{3}x$

$Opp(CPQS) = \frac{1}{2} \cdot x \cdot (5 + 2 \cdot \frac{4}{3}x + 5) = \frac{1}{2}x(10 + \frac{8}{3}x) = 16\frac{1}{2}$

$\left.\begin{array}{l} y_1 = \frac{1}{2}x(10 + \frac{8}{3}x) \\ y_2 = 16\frac{1}{2} \end{array}\right\}$ de optie intersect geeft: $x = 2,1$

dus de waterhoogte is: $4 + 2,1 = 6,1\,dm$

5.3 Wiskundige modellen opstellen

Opgave 29:

a. $x = 20$ en $h = 0$ geeft $400a + 20b = 0$

b. $x_{top} = 10$

$100a + 10b = 8$

c. $\begin{cases} 400a + 20b = 0 & \mid \times 1 \\ 100a + 10b = 8 & \mid \times 2 \end{cases}$

$\begin{cases} 400a + 20b = 0 \\ \underline{200a + 20b = 16} \quad - \end{cases}$

$200a = -16$

$a = -0,08$

$-8 + 10b = 8$

$10b = 16$

$b = 1,6$

Opgave 30:

$\begin{cases} 125a + 5b + 200 = 285 & \mid \times 8 \\ 512a + 8b + 200 = 648 & \mid \times 5 \end{cases}$

$\begin{cases} 1000a + 40b + 1600 = 2280 \\ \underline{2560a + 40b + 1000 = 3240} \quad - \end{cases}$

$-1560a + 600 = -960$

$-1560a = -1560$

$a = 1$

$125 + 5b + 200 = 285$

$5b = -40$

$b = -8$

Opgave 31:

$\begin{cases} 6,25a + 2,5b + 60000 = 40000 & \mid \times 2 \\ 25a + 5b + 60000 = 25000 & \mid \times 1 \end{cases}$

$\begin{cases} 12,5a + 5b + 120000 = 80000 \\ \underline{25a + 5b + 60000 = 25000} \quad - \end{cases}$

$-12,5a + 60000 = 55000$

$-12,5a = -5000$

$a = 400$

$10000 + 5b + 60000 = 25000$

$5b = -45000$

$b = -9000$

Opgave 32:

$$\begin{cases} 400a + 20b - 2200 = 80 & |\times 5 \\ 625a + 25b - 2200 = 89{,}5 & |\times 4 \end{cases}$$

$$\begin{cases} 2000a + 100b - 11000 = 400 \\ 2500a + 100b - 8800 = 358 \quad - \end{cases}$$

$-500a - 2200 = 42$

$-500a = 2242$

$a = -4{,}484$

$-1793{,}6 + 20b - 2200 = 80$

$20b = 4073{,}6$

$b = 203{,}68$

Opgave 33:

$$\begin{cases} 400a + 20b = 7{,}2 & |\times 3 \\ 900a + 30b = 9 & |\times 2 \end{cases}$$

$$\begin{cases} 1200a + 60b = 21{,}6 \\ 1800a + 60b = 18 \quad - \end{cases}$$

$-600a = 3{,}6$

$a = -0{,}006$

$-2{,}4 + 20b = 7{,}2$

$20b = 9{,}6$
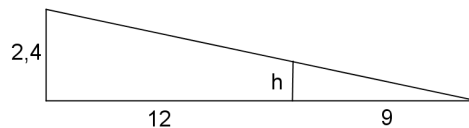
$b = 0{,}48$

Opgave 34:

a. $\dfrac{h}{9} = \dfrac{2{,}4}{21}$

$h = 1{,}03$

dus $h > 1$ voor $x = 12$ anders komt de bal in het net
als $h > 0$ voor $x = 21$ dan is de bal uit

b. $\begin{cases} 144a + 12b + 2{,}4 = 1{,}104 & |\times 5 \\ 400a + 20b + 2{,}4 = 0 & |\times 3 \end{cases}$

$$\begin{cases} 720a + 60b + 12 = 5{,}52 \\ 1200a + 60b + 7{,}2 = 0 \quad - \end{cases}$$

$-480a + 4{,}8 = 5{,}52$

$-480a = 0{,}72$

$a = -0{,}0015$

$-0{,}6 + 20b + 2{,}4 = 0$

$20b = -1{,}8$

$b = -0{,}09$

Opgave 35:

a. $I = 2x \cdot x \cdot h = 2x^2 h$

voor $x = 2$ geldt: $2 \cdot 2^2 \cdot h = 40$

$$8h = 40$$
$$h = 5\,\text{dm}$$

$M = 2 \cdot 2x \cdot h + 2 \cdot x \cdot h + 2x \cdot x = 6xh + 2x^2$

$M = 6 \cdot 2 \cdot 5 + 2 \cdot 2^2 = 68\,\text{dm}^2$

b. $2 \cdot 4^2 \cdot h = 40$

$32h = 40$

$h = 1{,}25$

$M = 6 \cdot 4 \cdot 1{,}25 + 2 \cdot 4^2 = 62\,\text{dm}^2$

c. $M = 6xh + 2x^2$

Opgave 36:

onderkant: $K = 0{,}4 \cdot 2x \cdot x = 0{,}8x^2$

voorkant: $K = 0{,}2 \cdot 2x \cdot h = 0{,}4xh$

rechterkant: $K = 0{,}2 \cdot x \cdot h = 0{,}2xh$

$K_{tot} = 0{,}8x^2 + 2 \cdot 0{,}4xh + 2 \cdot 0{,}2xh = 0{,}8x^2 + 1{,}2xh$

$I = 2x \cdot x \cdot h = 2x^2 h = 72$

$h = \dfrac{72}{2x^2} = \dfrac{36}{x^2}$

$K_{tot} = 0{,}8x^2 + 1{,}2x \cdot \dfrac{36}{x} = 0{,}8x^2 + \dfrac{43{,}2}{x}$

Opgave 37:

$I = x \cdot x \cdot h = 16$

$h = \dfrac{16}{x^2}$

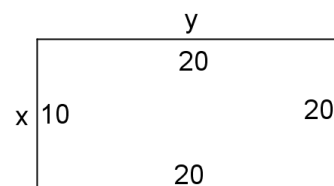$Opp = x^2 + 4xh = x^2 + 4x \cdot \dfrac{16}{x^2} = x^2 + \dfrac{64}{x}$

Opgave 38:

$Opp = xy = 75$

$y = \dfrac{75}{x}$

$K = 10x + 20y + 20x + 20y = 30x + 40y =$

$\qquad 30x + 40 \cdot \dfrac{75}{x} = 30x + \dfrac{3000}{x}$

Opgave 39:

$Opp = xy = 1200$

$y = \dfrac{1200}{x}$

$K = 15y + 15x + 60y = 15x + 75y = 15x + 75 \cdot \dfrac{1200}{x} = 15x + \dfrac{90000}{x}$

Opgave 40:

$I = \pi r^2 h = 100$

$h = \dfrac{100}{\pi r^2}$

$Opp = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi r \cdot \dfrac{100}{\pi r^2} = 2\pi r^2 + \dfrac{200}{r}$

$I = \pi r^2 h = 100$

5.4 Werken met wiskundige modellen

Opgave 41:
$x = 5$ geeft $h = 3$
$x = 25$ geeft $h = 0$

$\begin{cases} 25a + 5b = 3 & \bigg| \times 5 \\ 625a + 25b = 0 & \bigg| \times 1 \end{cases}$

$\begin{cases} 125a + 25b = 15 \\ \underline{625a + 25b = 0} \quad - \end{cases}$

$-500a = 15$
$a = -0,03$
$-0,75 + 5b = 3$
$5b = 3,75$
$b = 0,75$
$h = -0,03x^2 + 0,75x$
$h(12,5) = 4,69 \, \text{m}$

Opgave 42:

a. $\begin{cases} 9a + b = 25,4 \\ \underline{25a + b = 35} \quad - \end{cases}$

$-16a = -9,6$
$a = 0,6$
$5,4 + b = 25,4$
$b = 20$

b. $x = 0,6t^2 + 20$
$x(0) = 20$ dus 20 cm

c. $0,6t^2 + 20 = 80$
$0,6t^2 = 60$
$t^2 = 100$
$t = 10$
$y_1 = 0,6x^2 + 20$
$\left[\frac{dy}{dx}\right]_{x=10} = 12 \, ^{cm}\!/_{s}$

Opgave 43:

a. $h(200) = 0$
$h(200) = a \cdot (200 - p)^2 = 0$
$a \cdot (200 - p)^2 = 0$
$(200 - p)^2 = 0$
$p = 200$
$h(0) = a \cdot (0 - 200)^2 = 50$
$40000a = 50$
$a = 0,00125$

b. $0{,}00125(t-200)^2 = 25$

$\left.\begin{array}{r} y_1 = 0{,}00125(t-200)^2 \\ y_2 = 25 \end{array}\right\}$ de optie intersect geeft: $x = 58{,}6$

dus na 58,6 sec

c. $0{,}00125(t-200)^2 = 40$

$\left.\begin{array}{r} y_1 = 0{,}00125(t-200)^2 \\ y_2 = 40 \end{array}\right\}$ de optie intersect geeft: $x = 21{,}1$

$\left[\frac{dy}{dx}\right]_{x=21,1} = -0{,}45\,{}^{cm}\!/\!_s$

Opgave 44:

a. $\begin{cases} 8a + 4b = 80 \\ 1000a + 100b = 1200 \end{cases} \quad \begin{array}{l} \times 25 \\ \times 1 \end{array}$

$\begin{cases} 200a + 100b = 2000 \\ \underline{1000a + 100b = 1200} \quad - \end{cases}$

$-800a = 800$

$a = -1$

$-8 + 4b = 80$

$4b = 88$

$b = 22$

b. $y_1 = -x^3 + 22x^2$ optie maximium geeft: $x = 14{,}7 \quad \wedge \quad y = 1577$

dus op 16 september is het maximale aantal 1577

c. $\left.\begin{array}{r} y_1 = -x^3 + 22x^2 \\ y_2 = 1500 \end{array}\right\}$ optie intersect geeft: $x = 12{,}7 \quad \vee \quad x = 16{,}5$

dus gedurende 3,8 dagen

d. $\left[\frac{dy}{dx}\right]_{x=20} = -320$

Opgave 45:

a. $\begin{cases} 0{,}125a + 0{,}25b + 25 + 75 = 93 \\ 8a + 4b + 100 + 75 = 87 \end{cases} \quad \begin{array}{l} \times 16 \\ \times 1 \end{array}$

$\begin{cases} 2a + 4b + 1600 = 1488 \\ \underline{8a + 4b + 175 = 87} \quad - \end{cases}$

$-6a + 1425 = 1401$

$-6a = -24$

$a = 4$

$32 + 4b + 175 = 87$

$4b = -120$

$b = -30$

b. $y_1 = 4x^3 - 30x^2 + 50x + 75$ de optie minimum geeft: $x = 1{,}06$

dus na 63 minuten

c. $\left.\begin{array}{r} y_1 = 4x^3 - 30x^2 + 50x + 75 \\ y_2 = 95 \end{array}\right\}$ intersect geeft: $x = 0{,}597 \quad \vee \quad x = 1{,}572$

dus gedurende $60 \cdot (1{,}572 - 0{,}597) = 59$ minuten

d. $\left.\begin{array}{l} y_1 = 4x^3 - 30x^2 + 50x + 75 \\ y_2 = 70 \end{array}\right\}$ intersect geeft: $x = 2{,}70$

$\left[\frac{dy}{dx}\right]_{x=2,70} = -24{,}5\,°\!\!/\!_{uur}$

<u>Opgave 46:</u>

a. $\begin{cases} 1000a + 100b - 13 + 10 = 17 & | \times 4 \\ 8000a + 400b - 26 + 10 = 24 & | \times 1 \end{cases}$

$\begin{cases} 4000a + 400b - 12 = 68 \\ 8000a + 400b - 16 = 24 \quad - \end{cases}$

$-\,4000a + 4 = 44$

$-\,4000a = 40$

$a = -0{,}01$

$-\,10 + 100b - 3 = 17$

$100b = 30$

$b = 0{,}3$

b. $y_1 = -0{,}01x^3 + 0{,}3x^2 - 1{,}3x + 10$ de optie maximum geeft: $x = 17{,}53 \quad \wedge \quad y = 25{,}5$

dus maximaal $25{,}5°\,$C om 17.31 uur

c. $\left[\frac{dy}{dx}\right]_{x=12} = 1{,}58\,°\!\!/\!_{uur}$

d. $\left[\frac{dy}{dx}\right]_{x=22} = -2{,}62\,°\!\!/\!_{uur}$

<u>Opgave 47:</u>

a. $I = 1{,}5x \cdot x \cdot h = 48$

$h = \dfrac{32}{x^2}$

$Opp(onderkant) = 1{,}5x \cdot x = 1{,}5x^2$

$Opp(zijkanten) = 2 \cdot 1{,}5x \cdot h + 2 \cdot x \cdot h = 5xh = 5x \cdot \dfrac{32}{x^2} = \dfrac{160}{x}$

$K = 0{,}25 \cdot 1{,}5x^2 + 0{,}15 \cdot \dfrac{160}{x} = 0{,}375x^2 + \dfrac{24}{x}$

b. $y_1 = 0{,}375x^2 + \dfrac{24}{x}$ de optie minimum geeft: $x = 3{,}2$

dus $1{,}5x = 4{,}8$ en $h = \dfrac{32}{3{,}2^2} = 3{,}2$

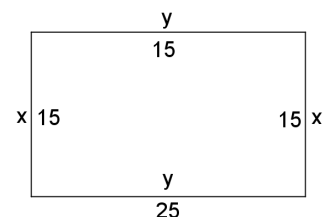dus de afmetingen zijn 48 bij 32 bij 32 cm

<u>Opgave 48:</u>

$Opp = xy = 1500$

$y = \dfrac{1500}{x}$

$K = 15x + 15y + 15x + 25y = 30x + 40y =$

$\qquad 30x + 40 \cdot \dfrac{1500}{x} = 30x + \dfrac{60000}{x}$

$y_1 = 30x + \dfrac{60000}{x}$ de optie minimum geeft: $x = 44{,}7$ dus $y = \dfrac{1500}{44{,}7} = 33{,}5$

Dus de afmetingen zijn 44,7 m bij 33,5 m

Opgave 49:

$Opp = xy = 2400$

$y = \dfrac{2400}{x}$

$K = 2x \cdot 40 + y \cdot 40 + 3x \cdot 15 = 125x + 40y = 125x + 40 \cdot \dfrac{2400}{x} = 125x + \dfrac{96000}{x}$

$y_1 = 125x + \dfrac{96000}{x}$ de optie minimum geeft $x = 27{,}7$ dus $y = \dfrac{2400}{27{,}7} = 86{,}6$

Dus de afmetingen zijn 86,6 m bij 27,7 m

Opgave 50:

a. $I = \pi r^2 h = 500$

      $h = \dfrac{500}{\pi r^2}$

      $K = \pi r^2 \cdot 1 + 2\pi r h \cdot 1 + 2\pi r \cdot 2 + \pi r^2 \cdot 2 =$

          $3\pi r^2 + 4\pi r + 2\pi r h =$

          $3\pi r^2 + 4\pi r + 2\pi r \cdot \dfrac{500}{\pi r^2} =$

          $3\pi r^2 + 4\pi r + \dfrac{1000}{r}$

b. $y_1 = 3\pi x^2 + 4\pi x + \dfrac{1000}{x}$ de optie minimum geeft: $x = 3{,}5$

      dus $r = 3{,}5$ cm en $h = \dfrac{500}{\pi \cdot 3{,}5^2} = 12{,}6$ cm

5.5 Diagnostische toets

Opgave 1:

a. $x = 0$ geeft: $3y = -17$

$\qquad y = -5\frac{2}{3}$

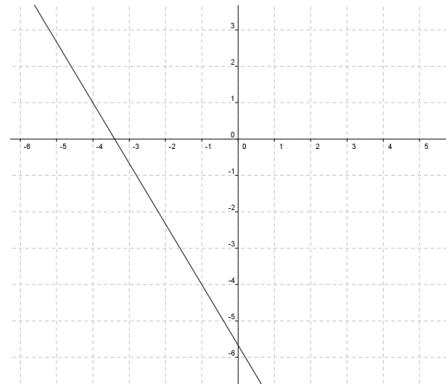$y = 0$ geeft: $5x = -17$

$\qquad x = -3\frac{2}{5}$

b. punt A: $5 \cdot -25 + 3 \cdot 36 = -17$ dus wel

punt B: $5 \cdot 60 + 3 \cdot -105 = -15$ dus niet

punt C: $5 \cdot 83 + 3 \cdot -144 = -17$ dus wel

c. $-280 + 3q = -17$

$3q = 263$

$q = 87\frac{2}{3}$

Opgave 2:

a. $\begin{cases} 2x - 5y = -9 \\ 3x + 5y = 24 \quad + \end{cases}$

$\quad 5x = 15$

$\quad x = 5$

$\quad 6 - 5y = -9$

$\quad -5y = -15$

$\quad y = 3$

b. $\begin{cases} 4x + 2y = 4 \\ 4x - 7y = 40 \quad - \end{cases}$

$\quad 9y = -36$

$\quad y = -4$

$\quad 4x - 8 = 4$

$\quad 4x = 12$

$\quad x = 3$

c. $\begin{cases} 4x + 5y = 27 & \big| \times 1 \\ -2x + 3y = 25 & \big| \times 2 \end{cases}$

$\begin{cases} 4x + 5y = 27 \\ -4x + 6y = 50 \quad + \end{cases}$

$\quad 11y = 77$

$\quad y = 7$

$\quad 4x + 35 = 27$

$\quad 4x = -8$

$\quad x = -2$

d. $\begin{cases} 6x - 3y = 19 & \big| \times 2 \\ -4x - 6y = 14 & \big| \times 1 \end{cases}$

$\begin{cases} 12x - 6y = 38 \\ -4x - 6y = 14 \quad - \end{cases}$

$$16x = 24$$
$$x = 1\tfrac{1}{2}$$
$$9 - 3y = 19$$
$$-3y = 10$$
$$y = -3\tfrac{1}{3}$$

Opgave 3:
$$\begin{cases} 5x - 6y = -27 & | \times 3 \\ 15x + 8y = 10 & | \times 1 \end{cases}$$

$$\begin{cases} 15x - 18y = -81 \\ 15x + 8y = 10 \quad - \end{cases}$$
$$-26y = -91$$
$$y = 3\tfrac{1}{2}$$
$$5x - 21 = -27$$
$$5x = -6$$
$$x = -1\tfrac{1}{5}$$
dus $S = (-1\tfrac{1}{5}, 3\tfrac{1}{2})$

Opgave 4:
a. $\begin{cases} 5x - 3y = 3 \\ y = \tfrac{2}{3}x - 4 \end{cases}$

$$5x - 3(\tfrac{2}{3}x - 4) = 3$$
$$5x - 2x + 12 = 3$$
$$3x = -9$$
$$x = -3$$
$$y = -6$$

b. $\begin{cases} x = 1,4y - 3 \\ -5x + 6y = 8 \end{cases}$

$$-5(1,4y - 3) + 6y = 8$$
$$-7y + 15 + 6y = 8$$
$$-y = -7$$
$$y = 7$$
$$x = 6,8$$

Opgave 5:
$$\begin{cases} 9a + 15 + c = -6 \\ a - 5 + c = -2 \quad - \end{cases}$$
$$8a + 20 = -4$$
$$8a = -24$$
$$a = -3$$
$$-3 - 5 + c = -2$$
$$c = 6$$

Opgave 6:

$$\begin{cases} -a + b = 4 \\ 5a + b = -8 \quad - \end{cases}$$
$$-6a = 12$$
$$a = -2$$
$$2 + b = 4$$
$$b = 2$$

$$\begin{cases} p + 4 = 4 \\ 25p + q = -8 \quad - \end{cases}$$
$$-24p = 12$$
$$p = -\tfrac{1}{2}$$
$$-\tfrac{1}{2} + q = 4$$
$$q = 4\tfrac{1}{2}$$

Opgave 7:
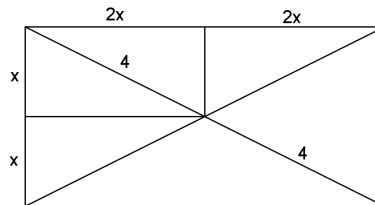
$$x^2 + (2x)^2 = 4^2$$
$$x^2 + 4x^2 = 16$$
$$5x^2 = 16$$
$$x^2 = 3,2$$
$$x = \sqrt{3,2}$$
$$l = 4x = 4\sqrt{3,2} = 7,2 \text{ cm}$$
$$b = 2x = 2\sqrt{3,2} = 3,6 \text{ cm}$$

Opgave 8:

$$\begin{cases} 125a + 5b + 1400 = 1250 & \mid \times 2 \\ 1000a + 10b + 1400 = 2600 & \mid \times 1 \end{cases}$$

$$\begin{cases} 250a + 10b + 2800 = 2500 \\ 1000a + 10b + 1400 = 2600 \quad - \end{cases}$$
$$-750a + 1400 = -100$$
$$-750a = -1500$$
$$a = 2$$
$$2000 + 10b + 1400 = 2600$$
$$10b = -800$$
$$b = -80$$

Opgave 9:

$$3l = 3000 \, cm^3$$
$$I = 3x \cdot x \cdot h = 3000$$

$$h = \frac{3000}{3x^2} = \frac{1000}{x^2}$$

$$Opp = 3x \cdot x + 2 \cdot 3x \cdot h + 2 \cdot x \cdot h = 3x^2 + 8xh = 3x^2 + 8x \cdot \frac{1000}{x^2} = 3x^2 + \frac{8000}{x}$$

Opgave 10:

a.
$$\begin{cases} a + b + 1 = 1,16 & \mid \times 4 \\ 8a + 4b + 2 = 2,48 & \mid \times 1 \end{cases}$$

$$\begin{cases} 4a + 4b + 4 = 4,64 \\ 8a + 4b + 2 = 2,48 \quad - \end{cases}$$
$$-4a + 2 = 2,16$$
$$-4a = 0,16$$
$$a = -0,04$$
$$-0,04 + b + 1 = 1,16$$
$$b = 0,2$$

b. $y_1 = -0,04x^3 + 0,2x^2 + x$

$\left[\frac{dy}{dx}\right]_{x=3} = 1,12$

c. de optie minimum geeft $x = 5$ dus na 5 maanden

Opgave 11:

a. $Opp = xy = 1200$

$$y = \frac{1200}{x}$$

$$l = 4x + y = 4x + \frac{1200}{x}$$

$y_1 = 4x + \dfrac{1200}{x}$ de optie minimum geeft $x = 17,3$ dus $y = \dfrac{1200}{17,3} = 69,3$

Dus de afmetingen zijn 17,3 m bij 69,3 m

b. $K = 60(2x + y) + 20 \cdot 2x$

$$= 120x + 60y + 40x$$
$$= 160x + 60y$$
$$= 160x + 60 \cdot \frac{1200}{x}$$
$$= 160x + \frac{72000}{x}$$

$y_1 = 160x + \dfrac{72000}{x}$ de optie minimum geeft: $x = 21,2$ dus $y = \dfrac{1200}{21,2} = 56,6$

Dus de afmetingen zijn 21,2 m bij 56,6 m